#### **ACM ICPC**

- Teams set?
- · Seven rooms reserved
- Checking on vans (2); need backup drivers
- Maintain your profile at Baylor site\*
- Need to set date/time for practice contest\* Remaining..
  - Friday, Oct 16 @ 6pm
  - Monday, Oct 19 @ 6pm
  - Friday, Oct 23 @ 6pm
  - Sunday, Oct 25@ 12pm

### **Upcoming Schedule**

- Oct 6 (today) Number Theory
- Oct 13 October break
- Oct 20 Backtracking
- Oct 27 Graph Traversal (Zhanibek)
- Nov 3 Graph Algorithms (Atallah?)
- Nov 10 Dynamic Programming
- Nov 17 Grids
- Nov 24 no class



<sup>\*</sup>See my homepage or Facebook group for links

# Today's Topic

- Number theory
- Concepts to know
  - Prime numbers
  - Divisibility
  - Modular arithmetic
  - Congruences

#### **Prime Numbers**

- Prime factorizations are unique "fundamental theorem of arithmetic"
- Finding primes by repeated division

## Divisibility

- Equivalent, for integers a and b...
  - a | b ("a divides b")
  - b % a == 0
  - a\*k == b, for some integer k
- Greatest Common Divisor, gcd
  - Euclid's algorithm
  - a \* x + b \* y = gcd(a,b)
- Least Common Multiple, lcm lcm(x, y) = x\*y/gcd(x, y)

#### Modular Arithmetic

- Allows some computations without bignums
- Useful identities

$$(x + y) \% n == ((x \% n) + (y \% n)) \% n$$
  
 $(x - y) \% n == ((x \% n) - (y \% n)) \% n$   
 $-x \% n == (n-x) \% n$   
 $(x * y) \% n == ((x \% n) * (y \% n)) \% n$ 

• Example: Finding the last digit of 2<sup>100</sup>

#### Congruences

- Alternative notation for modular arithmetic
- Equivalent, for integers a, b, and n

```
a \equiv b \pmod{n}

a \% n == b

n \mid (a - b)
```

- Addition, subtraction, multiplication "work" preserving modulus
- Division: OK to divide out common factors of all three numbers, a, b, and n

## **Solving Linear Congruences**

Given integers a, b, and n, solve
 ax = b (mod n)
 for x.

#### **Other Hints**

- Java: No unsigned, watch for long vs int
- BigInteger in Java includes
  - gcd
  - modPow
  - modInverse
  - isProbablePrime
- UNIX is your friend
  - factor command
  - bc arbitrary precision calculator

## Today's Problems

- 110701 Light, More Light
- 110702 Carmichael Numbers
  - "Presentation Error" bug (?) on PC site
- 110703 Euclid Problem
  - Use "extended Euclid algorithm", e.g., from text
- 110704 Factovisors

# Today's Problems -- Hints

- 110701 Light, More Light
  - Brute force or counting divisors = timeout
  - Need trick: no loops (!)
- 110702 Carmichael Numbers
  - BigInteger works, but not needed (try without)
- 110703 Euclid Problem
  - Use "extended Euclid algorithm", e.g., from text
- 110704 Factovisors
  - Factor m; count primes in n!